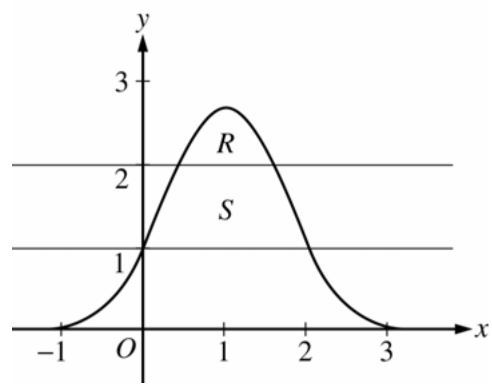
Question 1

Let R be the region bounded by the graph of $y = e^{2x-x^2}$ and the horizontal line $y = 2$, and let S be the region bounded by the graph of $y = e^{2x-x^2}$ and the horizontal lines $y = 1$ and $y = 2$, as shown above.

(a) Find the area of R.

(b) Find the area of S.

(c) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 1$.



$e^{2x-x^2} = 2$ when $x = 0.446057, 1.553943$

Let $P = 0.446057$ and $Q = 1.553943$

(a) Area of $R = \int_P^Q \left(e^{2x-x^2} - 2\right) dx = 0.514$

$3 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{limits} \\ 1 : \text{answer} \end{cases}$

(b) $e^{2x-x^2} = 1$ when $x = 0,\ 2$

Area of $S = \int_0^2 \left(e^{2x-x^2} - 1\right) dx -$ Area of R

$= 2.06016 -$ Area of $R = 1.546$

OR

$\int_0^P \left(e^{2x-x^2} - 1\right) dx + (Q - P) \cdot 1 + \int_Q^2 \left(e^{2x-x^2} - 1\right) dx$

$= 0.219064 + 1.107886 + 0.219064 = 1.546$

$3 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{limits} \\ 1 : \text{answer} \end{cases}$

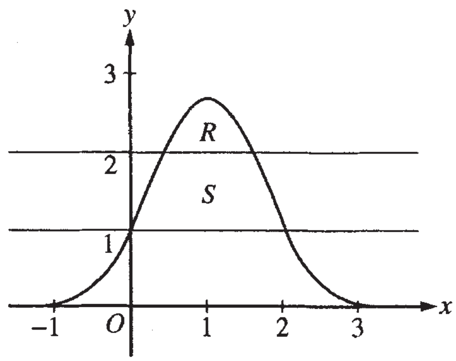
(c) Volume $= \pi \int_P^Q \left(\left(e^{2x-x^2} - 1\right)^2 - (2 - 1)^2\right) dx$

$3 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{constant and limits} \end{cases}$

1A1

CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.



Work for problem 1(a)

$e^{2x-x^2} = 2 \Rightarrow x = 0.446$ and $x = 1.554$

let $a = 0.446$ and $b = 1.554$

$$\text{Area} = \int_a^b e^{2x-x^2} - 2\,dx$$

$$= \int_{0.446}^{1.554} e^{2x-x^2} - 2\,dx$$

$$= 0.514 \text{ unit}^2$$

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Continue problem 1 on page 5.

1A₂

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Work for problem 1(b)

Area of S $= \int_0^2 e^{2x-x^2} - 1 \, dx$ − Area of R

$= 2.060 - 0.514$

$= 1.546 \text{ unit}^2$

Work for problem 1(c)

$$V = \pi \int_a^b \left(e^{2x-x^2} - 1\right)^2 - (2-1)^2 \, dx$$

$$\Rightarrow V = \pi \int_{0.446}^{1.554} \left(e^{2x-x^2} - 1\right)^2 - 1 \, dx$$

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1B,

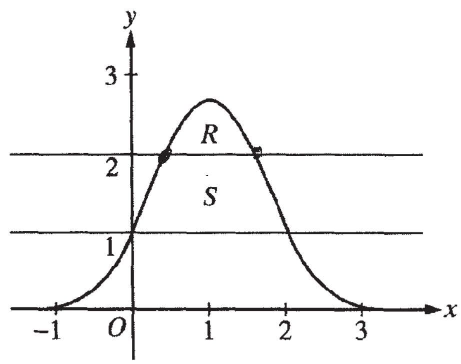
CALCULUS BC

SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.



Work for problem 1(a)

(a) $y = 2 = e^{2x-x^2}$

$\ln 2 = 2x - x^2 \Rightarrow x^2 - 2x + \ln 2 = 0$

$x = 1 \pm \sqrt{1 - \ln 2}$

$x = 1.554, 0.446$

$R = \int_{0.446}^{1.554} e^{2x-x^2}\,dx - 2\times[1.554 - 0.446]$

$= 2.730 - 2.216$

$= 0.514$

$\therefore R = 0.514$

Continue problem 1 on page 5.

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1B₂

Work for problem 1(b)

$y = e^{2x - x^2} = 1$

$\ln 1 = 2x - x^2$

$0 = x(2-x)$

$x = 2, 0$

$S = \int_0^2 e^{2x - x^2}\,dx - R - 2 \times 1$

$= 4.060 - 0.514 - 2$

$= 1.546$

$\therefore S = 1.546$

Work for problem 1(c)

$V = \int_0^2 \pi \left(e^{2x - x^2} - 1\right)^2 dx$

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CALCULUS AB

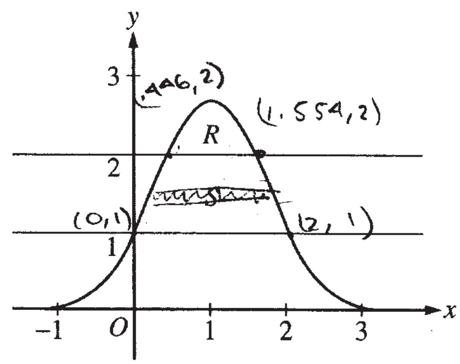
SECTION II, Part A

Time—45 minutes

Number of problems—3

1C1

A graphing calculator is required for some problems or parts of problems.



Work for problem 1(a)

a) Area R = $\int_{.446}^{1.554} \left(e^{2x-x^2} - 2\right) dx = 0.514$

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Continue problem 1 on page 5.

 1C2

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Work for problem 1(b)

Area S = $\int_{.446}^{1.554} (2.718 \cdot e^{2x-x^2}) - (e^{2x-x^2} - 1)\, dx =$

Work for problem 1(c)

$2\pi \int_{.446}^{1.554} (e^{2x-x^2} - 1)^2 \, dx$

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GO ON TO THE NEXT PAGE.

Question 1

Sample: 1A
Score: 9

The student earned all 9 points.

Sample: 1B
Score: 6

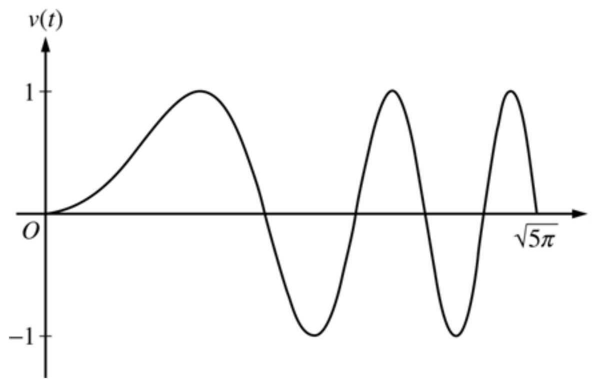
The student earned 6 points: 3 points in part (a), 3 points in part (b), and no points in part (c). Correct work is presented in parts (a) and (b). Although the student attempts a correct solution by rotating the region $R + S$ about $y = 1,$ the response does not subtract the volume obtained when region S is rotated about $y = 1.$ The integrand and the limits are incorrect, so the student did not earn any points in part (c).

Sample: 1C
Score: 3

The student earned 3 points: 3 points in part (a), no points in part (b), and no points in part (c). The student presents correct work in part (a). Incorrect limits and an incorrect integrand are shown in part (b), so no points were earned. In part (c) the student has an incorrect integrand and so did not earn the first 2 points. The correct limits are shown, but the student did not earn the limits and constant point because of the extra factor of 2 multiplied by the integral.

Question 2

A particle moves along the x-axis so that its velocity v at time $t \ge 0$ is given by $v(t) = \sin\left(t^2\right)$. The graph of v is shown above for $0 \le t \le \sqrt{5\pi}$. The position of the particle at time t is $x(t)$ and its position at time $t = 0$ is $x(0) = 5$.



(a) Find the acceleration of the particle at time $t = 3$.

(b) Find the total distance traveled by the particle from time $t = 0$ to $t = 3$.

(c) Find the position of the particle at time $t = 3$.

(d) For $0 \le t \le \sqrt{5\pi}$, find the time t at which the particle is farthest to the right. Explain your answer.

(a) $a(3) = v'(3) = 6\cos 9 = -5.466$ or -5.467

$1 : a(3)$

(b) Distance $= \int_0^3 |v(t)|\, dt = 1.702$

OR

For $0 < t < 3$, $v(t) = 0$ when $t = \sqrt{\pi} = 1.77245$ and $t = \sqrt{2\pi} = 2.50663$

$x(0) = 5$

$x\left(\sqrt{\pi}\right) = 5 + \int_0^{\sqrt{\pi}} v(t)\, dt = 5.89483$

$x\left(\sqrt{2\pi}\right) = 5 + \int_0^{\sqrt{2\pi}} v(t)\, dt = 5.43041$

$x(3) = 5 + \int_0^3 v(t)\, dt = 5.77356$

$\left|x\left(\sqrt{\pi}\right) - x(0)\right| + \left|x\left(\sqrt{2\pi}\right) - x\left(\sqrt{\pi}\right)\right| + \left|x(3) - x\left(\sqrt{2\pi}\right)\right| = 1.702$

$2 : \begin{cases} 1 : \text{setup} \\ 1 : \text{answer} \end{cases}$

(c) $x(3) = 5 + \int_0^3 v(t)\, dt = 5.773$ or 5.774

$3 : \begin{cases} 2 \begin{cases} 1 : \text{integrand} \\ 1 : \text{uses } x(0) = 5 \end{cases} \\ 1 : \text{answer} \end{cases}$

(d) The particle's rightmost position occurs at time $t = \sqrt{\pi} = 1.772$.

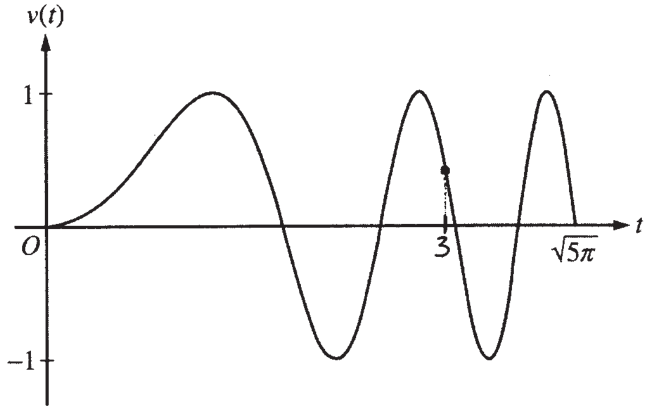
The particle changes from moving right to moving left at those times t for which $v(t) = 0$ with $v(t)$ changing from positive to negative, namely at $t = \sqrt{\pi}, \sqrt{3\pi}, \sqrt{5\pi}$ $(t = 1.772, 3.070, 3.963)$.

Using $x(T) = 5 + \int_0^T v(t)\, dt$, the particle's positions at the times it changes from rightward to leftward movement are:

T:	0	$\sqrt{\pi}$	$\sqrt{3\pi}$	$\sqrt{5\pi}$
$x(T)$:	5	5.895	5.788	5.752

The particle is farthest to the right when $T = \sqrt{\pi}$.

$3 : \begin{cases} 1 : \text{sets } v(t) = 0 \\ 1 : \text{answer} \\ 1 : \text{reason} \end{cases}$



Work for problem 2(a)

$a(t) = v'(t) = \cos(t^2)(2t)$
$= 2t\cos(t^2)$

$a(3) = 6\cos 9 = -5.467$

Work for problem 2(b)

The object reverses direction twice before $t = 3$.

$v(t) = \sin(t^2) = 0$
$t = \{1.772, 2.507\}$

Distance traveled $= \left|\int_0^{1.772} v(t)\,dt\right| + \left|\int_{1.772}^{2.507} v(t)\,dt\right| + \left|\int_{2.507}^{3} v(t)\,dt\right|$
$= 0.895 + 0.464 + 0.343$
$= 1.702$

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Continue problem 2 on page 7.

AB
2A2

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Work for problem 2(c)

Position at t=3 is the initial position plus net distance traveled.

$$x(3) = x(0) + \int_0^3 v(t)\,dt$$
$$= 5 + 0.774$$
$$= 5.774$$

Work for problem 2(d)

When the object reaches right and reverses direction, $x(t)$ has reached a relative maximum.

$\therefore$ $v(t) = 0$ for which $v(t)$ changes from +ve to −ve.

$t = \{1.772, 3.070, 3.963\}$

Comparing $x(t_1) = \int_0^{t_1} v(t)\,dt + 5$:

t_1	x
1.772	5.895
3.070	5.788
3.963	5.752

$\therefore$ As t increases, the maximum displacement to the right decreases.

$\therefore$ $x(t)$ has an absolute maximum at $t = 1.772$.

$\therefore$ The object is farthest right at $t = 1.772$.

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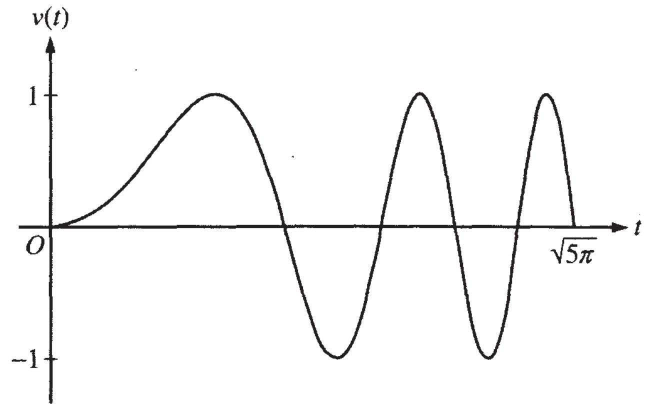
AB 2
2B1



Work for problem 2(a)

Acceleration: a(t) → rate of change of velocity = v'(t)

∴ $a(t) = (\sin(t^2))' = 2t\cos t$

∴ $a(3) = 2(3)\cdot\cos(3) = 6\cos 3 \approx -5.940$

Work for problem 2(b)

Total distance traveled accumulative velocity → $\int_a^b v(t)\,dt$

∴ Total distance traveled · $\int_0^{\sqrt{\pi}} \sin(t^2)\,dt - \int_{\sqrt{\pi}}^{\sqrt{2\pi}} \sin(t)\,dt + \int_{\sqrt{2\pi}}^{3} \sin(t^2)\,dt$

$= 1.702$

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Continue problem 2 on page 7.

AB2
2B$_2$

Work for problem 2(c)

velocity: v(t) → rate of change of position.

Because initial position = 5, the position at t=3 is

$5 + \int_0^3 \sin t^2 \, dt = 5.774$

Work for problem 2(d)

when $t = \sqrt{\pi}$, the value of $\int_0^t v(t)\,dt$ becomes the greatest

So, the particle is farthest to the right when $t = \sqrt{\pi}$

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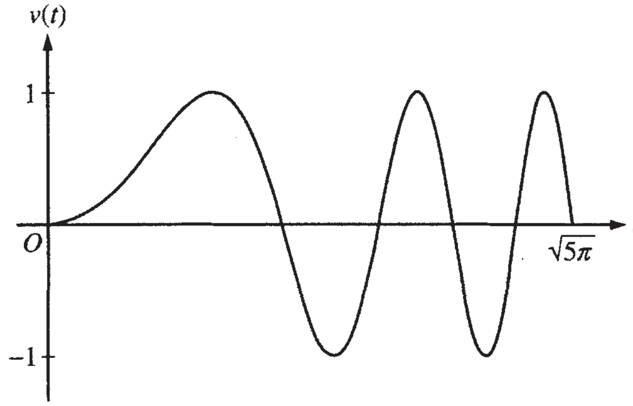
AB 2
2C1



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Work for problem 2(a)

$a = \frac{dv}{dt} = \frac{d \sin t^2}{dt} = 2t\cos t^2$

$a(3) = 6\cos 6^2 = -0.767$ unit/s²

the particle is decelerating

Work for problem 2(b)

$dis = x(t) = \int_0^3 8\ v(t) = \int_0^3 \sin(t^2)\ dt$

$= 0.774$ unit

Continue problem 2 on page 7.

AB2
2C/2

Work for problem 2(c)

$x(t) = \int_0 \sin(t^2)\, dt$

$x(t) = -\frac{1}{2t}\cos(t^2) + C \qquad x(0) = 5$

$5 = C$

$x(t) = -\frac{\cos t^2}{2t} + 5$

$x(3) = 5.152$ unit.

Work for problem 2(d)

It is farthest to the right at $\sqrt{\pi}$ as area above the graph is greater than the area below the graph.

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Question 2

Sample: 2A
Score: 9

The student earned all 9 points.

Sample: 2B
Score: 6

The student earned 6 points: no points in part (a), 2 points in part (b), 3 points in part (c), and 1 point in part (d). Correct work is presented in parts (b) and (c). In part (a) the student did not earn the first point because the derivative of $v(t)$ is incorrect. The student could have used the graphing calculator to find the numerical derivative. In part (d) the student does not set $v(t) = 0,$ so the first point was not earned. The answer point was earned but not the reason point since the student does not explicitly rule out the other times for which $v(t) = 0.$

Sample: 2C
Score: 3

The student earned 3 points: no points in part (a), no points in part (b), 2 points in part (c), and 1 point in part (d). In part (a) the derivative of $v(t)$ is correct, but the student makes an error when evaluating the acceleration at $t = 3.$ In part (b) the student integrates the velocity to find displacement instead of integrating the speed to find distance traveled. In this case, since the particle changes direction on the interval from $t = 0$ to $t = 3,$ displacement is not the same as distance traveled. In part (c) the student has a correct integrand and uses $x(0) = 5,$ which earned the first 2 points. The student attempts to find the antiderivative of $v(t)$ but did not earn the last point. In part (d) the student does not set $v(t) = 0,$ so the first point was not earned. The answer point was earned but not the reason point since the student does not explicitly rule out the other times when $v(t) = 0.$

Question 3

The wind chill is the temperature, in degrees Fahrenheit $(°\text{F})$, a human feels based on the air temperature, in degrees Fahrenheit, and the wind velocity v, in miles per hour (mph). If the air temperature is $32°\text{F}$, then the wind chill is given by $W(v) = 55.6 - 22.1v^{0.16}$ and is valid for $5 \le v \le 60$.

(a) Find $W'(20)$. Using correct units, explain the meaning of $W'(20)$ in terms of the wind chill.

(b) Find the average rate of change of W over the interval $5 \le v \le 60$. Find the value of v at which the instantaneous rate of change of W is equal to the average rate of change of W over the interval $5 \le v \le 60$.

(c) Over the time interval $0 \le t \le 4$ hours, the air temperature is a constant $32°\text{F}$. At time $t = 0$, the wind velocity is $v = 20$ mph. If the wind velocity increases at a constant rate of 5 mph per hour, what is the rate of change of the wind chill with respect to time at $t = 3$ hours? Indicate units of measure.

(a) $W'(20) = -22.1 \cdot 0.16 \cdot 20^{-0.84} = -0.285$ or -0.286 When $v = 20$ mph, the wind chill is decreasing at $0.286\ °\text{F}/\text{mph}$.	$2: \begin{cases} 1: \text{value} \\ 1: \text{explanation} \end{cases}$
(b) The average rate of change of W over the interval $5 \le v \le 60$ is $\dfrac{W(60) - W(5)}{60 - 5} = -0.253$ or -0.254. $W'(v) = \dfrac{W(60) - W(5)}{60 - 5}$ when $v = 23.011$.	$3: \begin{cases} 1: \text{average rate of change} \\ 1: W'(v) = \text{average rate of change} \\ 1: \text{value of } v \end{cases}$
(c) $\left.\dfrac{dW}{dt}\right\|_{t=3} = \left.\left(\dfrac{dW}{dv} \cdot \dfrac{dv}{dt}\right)\right\|_{t=3} = W'(35) \cdot 5 = -0.892\ °\text{F}/\text{hr}$ OR $W = 55.6 - 22.1(20 + 5t)^{0.16}$ $\left.\dfrac{dW}{dt}\right\|_{t=3} = -0.892\ °\text{F}/\text{hr}$	$3: \begin{cases} 1: \dfrac{dv}{dt} = 5 \\ 1: \text{uses } v(3) = 35, \\ \quad\quad \text{or} \\ \quad \text{uses } v(t) = 20 + 5t \\ 1: \text{answer} \end{cases}$
Units of $°\text{F}/\text{mph}$ in (a) and $°\text{F}/\text{hr}$ in (c)	1 : units in (a) and (c)

Work for problem 3(a)

$W(v) = 55.6 - 22.1\,V^{0.16}$

$W'(v) = -22.1(0.16)\,V^{0.16-1}$

$= -3.536\,V^{-0.84}$

$W'(20) \approx -3.536(20)^{-0.84}$

$\approx -0.286\ {}^\circ F/mph$

It means that the wind chill is decreasing at a rate of 0.286 °F/mph when V=20 mph.

Work for problem 3(b)

$W'(v) = -3.536\,V^{-0.84}$

avg. rate of change of W

$= \frac{1}{60-5}\int_5^{60} W'(v)\,dV$

$= \frac{1}{55}\int_5^{60} -3.536\,V^{-0.84}\,dV$

$= \frac{1}{55}(-13.95882)$

$\approx -0.254\ {}^\circ F/mph$

$W'(v) = -0.254$

$-3.536V^{-0.84} = -0.254$

$V = 22.989$ mph

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Continue problem 3 on page 9.

3A2

Work for problem 3(c)

$\frac{dv}{dt} = 5$

$\int dv = \int 5\, dt$

$V = 5t + c$

$20 = 5(0) + c$

$c = 20$

$v(t) = 5t + 20$

@ $t = 3$.

$v(3) = 15 + 20 = 35$ mph.

$\frac{dw}{dv} = -3.536\, v^{-0.84}$

$\frac{dv}{dt} = 5$

$\frac{dw}{dt} = \frac{dw}{dv} \cdot \frac{dv}{dt}$

$= (-3.536\, v^{-0.84})(5)$

$\frac{dw}{dt}\Big|_{t=3} = \left[-3.536 (35)^{-0.84}\right](5)$

≈ -0.892 °F/h

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END OF PART A OF SECTION II
IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

Work for problem 3(a)

$W(v) = 55.6 - 22.1v^{.16}$

$W'(v) = -22.1(.16)v^{-.84}$

$W'(20) = -.286$ °F/mph the rate of change of the windchill at $(v=20)$

Work for problem 3(b)

$$\frac{F(60)-F(5)}{60-5} = \frac{13.0503-27.0091}{60-5} = \boxed{-.254}$$

$W'(v) = -22.1(.16)v^{-.84} = -.254$

$\frac{1}{-.84}(v^{-.84}) = (.7177)^{1/-.84}$

$\boxed{v = 23.011}$

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Continue problem 3 on page 9.

Work for problem 3(c)

$W(v) = 55.6 - 22.1v^{.16}$

$w'(v) = -22.1(.16)v^{-.84}$

$w'(35) = -.178 \frac{\text{degrees}}{\text{mph}}$

$\frac{dv}{dt} = 5$

$v @ t=3 = 35$

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END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

Work for problem 3(a)

$W'(v) = .16(-22.1)v^{-.84}$ $v = 20$

$W'(20) = .16(-22.1)(20^{-.84})$

$= -.2855$

$\boxed{W'(20) = -.286 \text{ m/hr.}}$

W'(20) is how fast and in which direction the wind chill is moving when the air temperature is felt when the wind is traveling at a velocity of 20 mph

Work for problem 3(b)

average rate change $= \dfrac{W(b) - W(a)}{b-a}$

$= \dfrac{W(60) - W(5)}{b-a}$

$= \dfrac{(55.6 - 22.1(60)^{.16}) - (55.6 - 22.1(5)^{.16})}{60-5}$

$\dfrac{13.0503 - 27.0091}{55}$

$= -.2537$

avg rate of chang $\boxed{= -.254 \text{ m/hr}}$

Continue problem 3 on pa

 3C2

Work for problem 3(c)

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$\frac{dV}{dt} = 5$ m/me $t=0$
$V=20$

$V = 20 + 5t$ $x = 3$

@ $V_{t=3} = 20 + 15$
$= 45$

$$\frac{W(45) - W(20)}{45 - 20}$$

END OF PART A OF SECTION II
IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

Question 3

Sample: 3A
Score: 9

The student earned all 9 points. The answer of 22.989 in part (b) is acceptable. In this case the student sets $W'(v)$ equal to the correct average rate of change rounded to three decimal places and correctly solves for v.

Sample: 3B
Score: 6

The student earned 6 points: 1 point in part (a), 3 points in part (b), 2 points in part (c), and no units point. In part (a) $W'(20)$ is correct, but the student does not give a complete explanation. It was necessary for the student to appeal to the fact that the wind chill is decreasing and not merely changing. In part (b) the student calls the function F instead of W but correctly finds the average rate of change. In part (c) the student earned the first 2 points but does not apply the chain rule to come up with the required answer. The student does not use correct units.

Sample: 3C
Score: 3

The student earned 3 points: 1 point in part (a), 1 point in part (b), 1 point in part (c), and no units point. In part (a) $W'(20)$ is correct, but the student does not explain that the wind chill is decreasing. In part (b) the student earned the first point for the average rate of change. In part (c) the student earned the first point but makes a mistake in calculating the velocity at $t = 3,$ so the second point was not earned. Although the student was eligible for the third point, it was not earned since $\frac{dW}{dt}$ was not found at $t = 3.$ The student does not use correct units.

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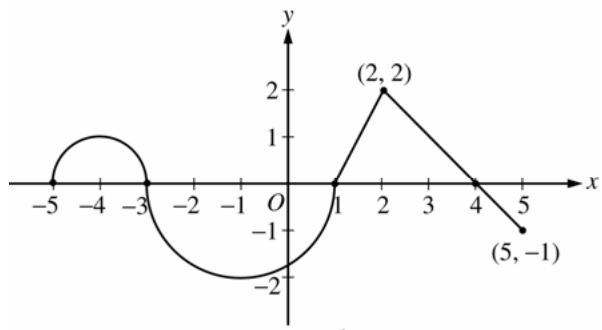
Question 4

Let f be a function defined on the closed interval $-5 \le x \le 5$ with $f(1) = 3$. The graph of f', the derivative of f, consists of two semicircles and two line segments, as shown above.



Graph of f'

(a) For $-5 < x < 5$, find all values x at which f has a relative maximum. Justify your answer.

(b) For $-5 < x < 5$, find all values x at which the graph of f has a point of inflection. Justify your answer.

(c) Find all intervals on which the graph of f is concave up and also has positive slope. Explain your reasoning.

(d) Find the absolute minimum value of $f(x)$ over the closed interval $-5 \le x \le 5$. Explain your reasoning.

(a) $f'(x) = 0$ at $x = -3,\ 1,\ 4$ f' changes from positive to negative at -3 and 4. Thus, f has a relative maximum at $x = -3$ and at $x = 4$.	$2 : \begin{cases} 1 : x\text{-values} \\ 1 : \text{justification} \end{cases}$
(b) f' changes from increasing to decreasing, or vice versa, at $x = -4,\ -1,$ and 2. Thus, the graph of f has points of inflection when $x = -4,\ -1,$ and 2.	$2 : \begin{cases} 1 : x\text{-values} \\ 1 : \text{justification} \end{cases}$
(c) The graph of f is concave up with positive slope where f' is increasing and positive: $-5 < x < -4$ and $1 < x < 2$.	$2 : \begin{cases} 1 : \text{intervals} \\ 1 : \text{explanation} \end{cases}$
(d) Candidates for the absolute minimum are where f' changes from negative to positive (at $x = 1$) and at the endpoints ($x = -5, 5$). $f(-5) = 3 + \int_1^{-5} f'(x)\,dx = 3 - \frac{\pi}{2} + 2\pi > 3$ $f(1) = 3$ $f(5) = 3 + \int_1^{5} f'(x)\,dx = 3 + \frac{3 \cdot 2}{2} - \frac{1}{2} > 3$ The absolute minimum value of f on $[-5, 5]$ is $f(1) = 3$.	$3 : \begin{cases} 1 : \text{identifies } x = 1 \text{ as a candidate} \\ 1 : \text{considers endpoints} \\ 1 : \text{value and explanation} \end{cases}$

NO CALCULATOR ALLOWED

4A1

CALCULUS AB

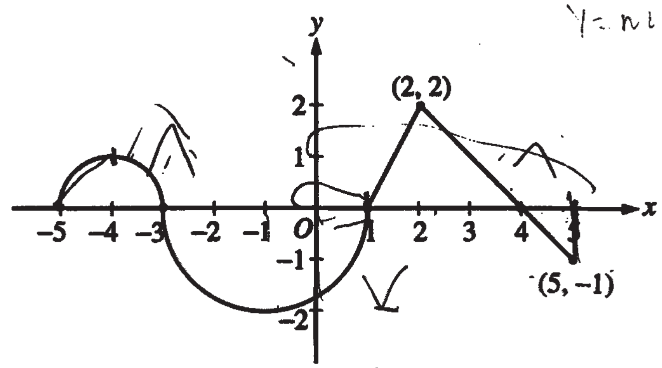
SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.

Y=mx+b



Graph of f'

Work for problem 4(a)

a) relative maximum at $x = -3,\ 4$

At $x=-3,\ 4$, the graph of f' change from positive to negative, which hints the graph of f change from increase to decrease. so at $x=-3,\ 4$, f has relative maximums

Work for problem 4(b)

points of inflection at. $x=-4, -1, 2$

at all these x points, the graph of f' change from increase to decreace or from decrease to increase, which hints at these points, f change from concave up to concave down or concave down to concave up,

Do not write beyond this border.

Continue problem 4 on page 11.

4A2

Do not write beyond this border.

Work for problem 4(c) when $-5 < x < -4$, $1 < x < 2$,
the graph of f is concave up and also has positive slope.
From the graph of f', when $-5 < x < -4$ and $1 < x < 2$,
the graph of f' is both increasing and above x-axis,
which shows f' and f'' are both positive.
positive f' means the slope of f is positive
and positive f'' means f is concave upward.

Work for problem 4(d) From the graph of f', the only local minimum of f is at $x=1$., $f(1)=3$

$$\int_{-5}^{5} f'(x)\,dx = F(5) - F(-5) = 2\pi - 8\pi + 3 - \frac{1}{2}$$
$$= \frac{5}{2} - 6\pi < 0$$

so $F(5) < F(-5)$

$$\int_{1}^{5} f'(x)\,dx = F(5) - F(1) = \frac{3\times 2}{2} - \frac{1}{2} = \frac{5}{2} > 0$$

so $F(5) > F(1)$

thus the absolute minimum value of $f(x)$ over the close interval $-5 \le x \le 5$. is 3.

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NO CALCULATOR ALLOWED

4B1

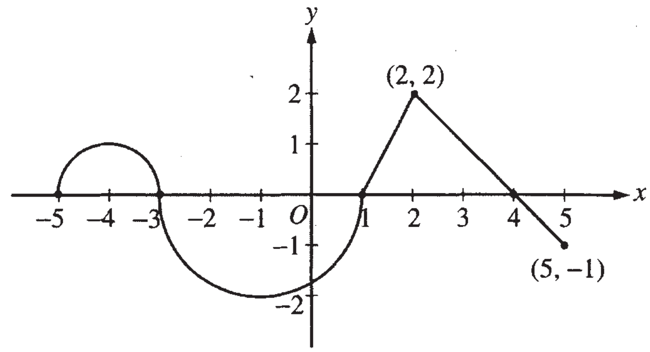
CALCULUS AB

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.



Graph of f'

Work for problem 4(a)

~~at $x = -4$, and $x = 2$~~

~~at these points, f' changes from increasing to decreasing~~

at $x = -3$, $x = 4$

at these points f' changes from positive to negative

Work for problem 4(b)

at $x = -4$, $x = -1$,

at these points f' changes from increasing to decreasing

Continue problem 4 on page 11.

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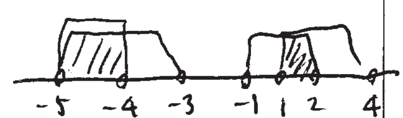
Work for problem 4(c)

f is concave up and has positive slope when $f''(x)>0$ and $f'(x)>0$

$f''(x)>0$ means the slope of f' is positive.

so $f''(x)>0$ when $(-5,4)$, $(-1,2)$,

$f'(x)>0$ when $(-5,-3)$, $(1,4)$,



the intervals are $(-5,-4)$, $(1,2)$

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Work for problem 4(d)

$x^2+y^2=1$ $\quad y^2=1-x^2$ $\quad y=\sqrt{1-x^2}$

$f(x)$ is minimum at the endpoints or at $x=1$ because f' changes from negative to positive at $x=1$.

$f(-5)=$

$f(1)=3$

$f(5)=-\frac{5^2}{2}+4\cdot 5-\frac{1}{2}=-14,$

$\frac{2+1}{2-5}=-1$ $\quad y-2=-(x-2)$ $\quad f(1)=-\frac{1}{2}+4+C=3$

$C=-\frac{1}{2}$

$y=-x+4,$ $\quad f(x)=\int(-x+4)\,dx=-\frac{x^2}{2}+4x+C$

$\frac{-25}{2}+\frac{40}{2}-\frac{1}{2}=-14$

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NO CALCULATOR ALLOWED

4C1

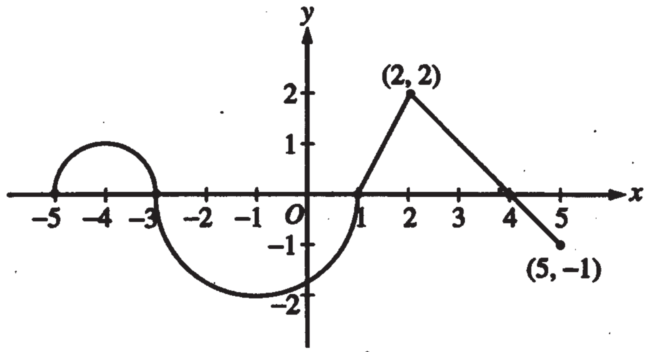
CALCULUS AB

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.



Graph of f'

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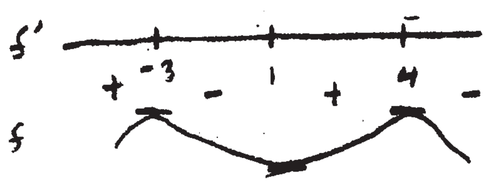
Work for problem 4(a)



$f'(x)$ changes sign at
$x = -3, 1, 4$
changes sign from $-$ to $+$ at $x = -3, 4$

$\therefore \quad x = -3, 4$

Work for problem 4(b)

point of inflection occur when $f''(x) = 0$ or is undefined.

$\therefore \ x = -4, -3, -1, 1, 2$

Continue problem 4 on page 11.

4C₂

Work for problem 4(c)

f': -3, 1, 4
$+ \quad - \quad + \quad -$

f'': -4, -3, -1, 1, 2
$+ \quad - \quad - \quad + \quad + \quad -$

$\therefore \ (-5,-4),\ (1,2)$

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Work for problem 4(d)

$\frac{1}{2}\pi - \frac{1}{2}4\pi = -\frac{3}{2}\pi$

$f'(x) < 0$ at $-3 < x < 1$
so
$f(x)$ decrease at $-3 < x < 1$

$f'(x) < 0$ also at $4 < x < 5$
but

$\left| \int_1^4 f(x) \right| > \left| \int_4^5 f'(x) \right|$

there fore
$f(x)$ have its absolute minimum value at $x = 1$

$\boxed{\therefore \ -\frac{3}{2}\pi}$

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AP® CALCULUS AB
2007 SCORING COMMENTARY (Form B)

Question 4

Sample: 4A
Score: 9

The student earned all 9 points.

Sample: 4B
Score: 6

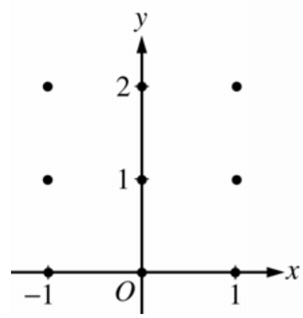
The student earned 6 points: 2 points in part (a), no points in part (b), 2 points in part (c), and 2 points in part (d). Correct work is presented in parts (a) and (c). In part (b) the student only finds two of the three values, so the first point was not earned. The justification point was not earned because it is not true that f' changes from increasing to decreasing at $x = -1$. In part (d) the student earned the first 2 points since $x = 1$ is identified as a candidate and the endpoints are considered. Since the student never concludes that the absolute minimum is 3, the third point was not earned.

Sample: 4C
Score: 4

The student earned 4 points: 2 points in part (a), no points in part (b), 1 point in part (c), and 1 point in part (d). Correct work is presented in part (a). In part (b) the student gives two additional, incorrect values, so the first point was not earned. No justification is included. In part (c) the first point is earned because of the correct intervals. The student's sign chart alone did not earn the explanation point. It was necessary to explain the reasoning from the sign chart. In part (d) the student earned the first point since $x = 1$ is identified as a candidate. The student does not consider both endpoints and does not give a correct answer, so the last 2 points were not earned.

Question 5

Consider the differential equation $\frac{dy}{dx} = \frac{1}{2}x + y - 1$.



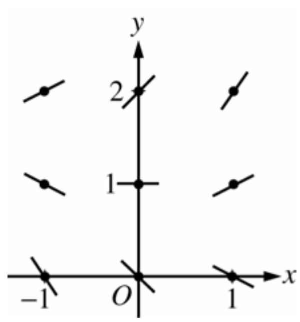
(a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.
(Note: Use the axes provided in the exam booklet.)

(b) Find $\frac{d^2y}{dx^2}$ in terms of x and y. Describe the region in the xy-plane in which all solution curves to the differential equation are concave up.

(c) Let $y = f(x)$ be a particular solution to the differential equation with the initial condition $f(0) = 1$. Does f have a relative minimum, a relative maximum, or neither at $x = 0$? Justify your answer.

(d) Find the values of the constants m and b, for which $y = mx + b$ is a solution to the differential equation.

(a)



2 : Sign of slope at each point and relative steepness of slope lines in rows and columns.

(b) $\frac{d^2y}{dx^2} = \frac{1}{2} + \frac{dy}{dx} = \frac{1}{2}x + y - \frac{1}{2}$

Solution curves will be concave up on the half-plane above the line $y = -\frac{1}{2}x + \frac{1}{2}$.

$$3 : \begin{cases} 2 : \frac{d^2y}{dx^2} \\ 1 : \text{description} \end{cases}$$

(c) $\left.\frac{dy}{dx}\right|_{(0,\,1)} = 0 + 1 - 1 = 0$ and $\left.\frac{d^2y}{dx^2}\right|_{(0,\,1)} = 0 + 1 - \frac{1}{2} > 0$

Thus, f has a relative minimum at $(0, 1)$.

$$2 : \begin{cases} 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$$

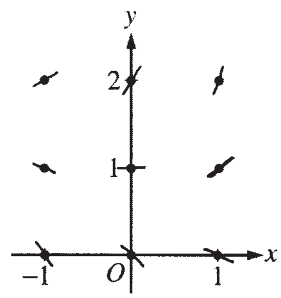
(d) Substituting $y = mx + b$ into the differential equation:

$m = \frac{1}{2}x + (mx + b) - 1 = \left(m + \frac{1}{2}\right)x + (b - 1)$

Then $0 = m + \frac{1}{2}$ and $m = b - 1$: $m = -\frac{1}{2}$ and $b = \frac{1}{2}$.

$$2 : \begin{cases} 1 : \text{value for } m \\ 1 : \text{value for } b \end{cases}$$

Work for problem 5(a)



Work for problem 5(b)

$$\frac{d^2y}{dx^2} = \frac{1}{2} + \frac{dy}{dx} = \frac{1}{2}x + y - \frac{1}{2}$$

curves are concave up : $\frac{d^2y}{dx^2} > 0$

$\frac{1}{2}x + y - \frac{1}{2} > 0$

$\frac{1}{2}x + y > \frac{1}{2}$

$x + 2y > 1$

$\boxed{x + 2y - 1 > 0}$ of the solution curves

when coordinates satisfy this condition, the solution curves are concave up.

... $x \geq 0$ and $y > \frac{1}{2}$

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Continue problem 5 on page 13.

AB
5A2

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Work for problem 5(c)

$f(0) = 1$

$\frac{dy}{dx} = \frac{1}{2} \cdot 0 + 1 - 1 = 0$

$\frac{d^2y}{dx^2} = \frac{1}{2} \cdot 0 + 1 - \frac{1}{2} = \frac{1}{2} > 0$

f has a relative minimum at $x = 0$

as $\frac{dy}{dx}$ attains zero and change its sign from negative to positive.

Work for problem 5(d)

$y = mx + b$

$\frac{dy}{dx} = m = \underbrace{\frac{1}{2}x + y} - 1 \rightarrow m = \frac{1}{2} - 1 = -\frac{1}{2}$

$\uparrow$

$\frac{d^2y}{dx^2} = 0 = \underbrace{\frac{1}{2}x + y} - \frac{1}{2} \rightarrow \underbrace{\frac{1}{2}x + y} = \frac{1}{2}$

$$\begin{cases} y = -\frac{1}{2}x + b \rightarrow \underbrace{2y} = -x + 2b \\ \frac{1}{2}x + y = \frac{1}{2} \rightarrow x + \underbrace{2y} = 1 \rightarrow \cancel{x - x} + 2b = 1 \end{cases}$$

$b = \frac{1}{2}$

$m = -\frac{1}{2}, \quad b = \frac{1}{2}$

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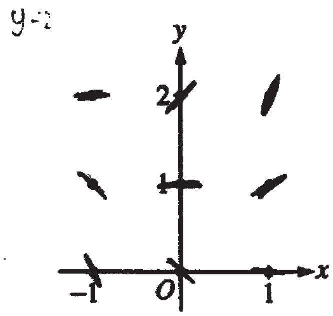
Work for problem 5(a)



Work for problem 5(b)

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(\frac{1}{2}x+y-1\right) = \frac{1}{2}+\frac{dy}{dx} = \frac{1}{2}x+y-\frac{1}{2}$$

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NO CALCULATOR ALLOWED

AB
5B$_2$

Work for problem 5(c)

$y =$

If $f(0)=1$, when $x=0$, $y=1$.

$\Rightarrow \frac{dy}{dx} = \frac{1}{2}x+y-1 = \frac{1}{2}\cdot 0+1-1 = 0$

$\therefore$ f have neither relative maximum and relative minimum values.

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Work for problem 5(d)

$y = -\frac{1}{2}x+b$

$y = mx+b$

$\frac{dy}{dx} = m = \frac{1}{2}x+y-1 =$

$\frac{d^2y}{dx^2} = 0 = \frac{1}{2}x+y-\frac{1}{2} = m+\frac{1}{2}$ $\quad \therefore m = -\frac{1}{2}$

$y = -\frac{1}{2}x+b \Rightarrow b = \frac{1}{2}x+y = m+1 = -\frac{1}{2}+1 = \frac{1}{2}$ $\quad \therefore b = \frac{1}{2}$

$\therefore y = -\frac{1}{2}x+\frac{1}{2}$ $\left(m=-\frac{1}{2}, b=\frac{1}{2}\right)$

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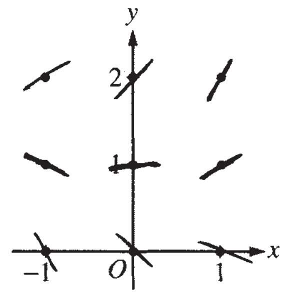
GO ON TO THE NEXT PAGE.

NO CALCULATOR ALLOWED

AB
5C$_1$

Work for problem 5(a)



Work for problem 5(b)

$$\frac{d}{dx}\ \tfrac{1}{2}x + y - 1 = \tfrac{1}{2} + \frac{dy}{dx}$$

$$\frac{d^2y}{dx^2} = \tfrac{1}{2}x + y - \tfrac{1}{2}$$

The First quadrent (x+, y+) (excluding the origin) is all concave up

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Continue problem 5 on page 13.

AB
$5C_2$

Work for problem 5(c)

neither

At $x=\emptyset$ the slope has a. range from neg to pos

Work for problem 5(d)

$$y=(\tfrac{1}{2}x+y-1)x+b$$

$$y=\tfrac{1}{2}x^2+xy-x+b$$

$$-xy \qquad -xy$$

$$\frac{y(1-x)}{1-x}=\frac{\tfrac{1}{2}x^2-x+b}{1-x}$$

$$y=\frac{\tfrac{1}{2}x^2-x+b}{1-x}$$

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Do not write beyond this border.

Question 5

Sample: 5A
Score: 9

The student earned all 9 points.

Sample: 5B
Score: 6

The student earned 6 points: 2 points in part (a), 2 points in part (b), no points in part (c), and 2 points in part (d). Correct work is presented in parts (a) and (d). In part (b) the student earned the first 2 points but gives no description of the region, so the third point was not earned. In part (c) the student does not conclude a relative minimum or provide a justification, so no points were earned. The student finds $\frac{dy}{dx}$ but does not calculate the second derivative at $(0, 1)$ to determine the concavity of the graph.

Sample: 5C
Score: 4

The student earned 4 points: 2 points in part (a), 2 points in part (b), no points in part (c), and no points in part (d). Correct work is presented in part (a). In part (b) the student earned the first 2 points but gives an incorrect description of the region, so the third point was not earned. There is no relevant work provided for part (c). In part (d) the student does not find the values of m or b.

AP® CALCULUS AB
2007 SCORING GUIDELINES (Form B)

Question 6

Let f be a twice-differentiable function such that $f(2) = 5$ and $f(5) = 2$. Let g be the function given by $g(x) = f(f(x))$.

(a) Explain why there must be a value c for $2 < c < 5$ such that $f'(c) = -1$.

(b) Show that $g'(2) = g'(5)$. Use this result to explain why there must be a value k for $2 < k < 5$ such that $g''(k) = 0$.

(c) Show that if $f''(x) = 0$ for all x, then the graph of g does not have a point of inflection.

(d) Let $h(x) = f(x) - x$. Explain why there must be a value r for $2 < r < 5$ such that $h(r) = 0$.

(a) The Mean Value Theorem guarantees that there is a value c, with $2 < c < 5$, so that $f'(c) = \dfrac{f(5) - f(2)}{5 - 2} = \dfrac{2 - 5}{5 - 2} = -1.$	$2: \begin{cases} 1: \dfrac{f(5) - f(2)}{5 - 2} \\ 1: \text{conclusion, using MVT} \end{cases}$
(b) $g'(x) = f'(f(x)) \cdot f'(x)$ $g'(2) = f'(f(2)) \cdot f'(2) = f'(5) \cdot f'(2)$ $g'(5) = f'(f(5)) \cdot f'(5) = f'(2) \cdot f'(5)$ Thus, $g'(2) = g'(5)$. Since f is twice-differentiable, g' is differentiable everywhere, so the Mean Value Theorem applied to g' on $[2, 5]$ guarantees there is a value k, with $2 < k < 5$, such that $g''(k) = \dfrac{g'(5) - g'(2)}{5 - 2} = 0.$	$3: \begin{cases} 1: g'(x) \\ 1: g'(2) = f'(5) \cdot f'(2) = g'(5) \\ 1: \text{uses MVT with } g' \end{cases}$
(c) $g''(x) = f''(f(x)) \cdot f'(x) \cdot f'(x) + f'(f(x)) \cdot f''(x)$ If $f''(x) = 0$ for all x, then $g''(x) = 0 \cdot f'(x) \cdot f'(x) + f'(f(x)) \cdot 0 = 0$ for all x. Thus, there is no x-value at which $g''(x)$ changes sign, so the graph of g has no inflection points. OR If $f''(x) = 0$ for all x, then f is linear, so $g = f \circ f$ is linear and the graph of g has no inflection points.	$2: \begin{cases} 1: \text{considers } g'' \\ 1: g''(x) = 0 \text{ for all } x \end{cases}$ OR $2: \begin{cases} 1: f \text{ is linear} \\ 1: g \text{ is linear} \end{cases}$
(d) Let $h(x) = f(x) - x$. $h(2) = f(2) - 2 = 5 - 2 = 3$ $h(5) = f(5) - 5 = 2 - 5 = -3$ Since $h(2) > 0 > h(5)$, the Intermediate Value Theorem guarantees that there is a value r, with $2 < r < 5$, such that $h(r) = 0$.	$2: \begin{cases} 1: h(2) \text{ and } h(5) \\ 1: \text{conclusion, using IVT} \end{cases}$

AB

6A₁

Work for problem 6(a)

$f'(c) = -1$ interval $= (2,5)$

$$\frac{f(5) - f(2)}{5-2} = \frac{2-5}{3} = -1$$

$f'(c) = -1$

According to the Mean Value Theorem, there must exist some c, such that $f'(c) = -1$

Work for problem 6(b)

$g(x) = f(f(x))$

$g'(x) = f'(f(x))f'(x)$ ← Chain Rule

$g'(2) = f'(f(2))f'(2)$

$g'(2) = f'(5)f'(2)$

$g'(5) = f'(f(5))f'(5)$

$g'(5) = f'(2)f'(5$

$f'(5)f'(2) = f'(2)f'(5)$

$\therefore g'(2) = g'(5)$

Mean Value Theorem: $\dfrac{g'(5) - g'(2)}{5-2} = \dfrac{0}{3} = 0$, therefore, there must exist some k within $2 < k < 5$ where $g''(k) = 0$

Work for problem 6(c)

$f''(x)=0$ for all x

pt of inflection on g is where $g''=0$

$g''(x) = f''(f'(f(x)))f'(x) + f'(f(x))f''(x)$

If $f''=0$, then

$g''(x) = 0 + 0$

$= 0$, for every x, meaning there is no point on g where the graph changes concavity.

Work for problem 6(d)

$h(x) = f(x) - x$

$(2,5)$

$h(2) = f(2) - 2$

$= 5 - 2 = \underline{\underline{3}}$

$h(5) = f(5) - 5$

$= 2 - 5 = \underline{\underline{-3}}$

Because the values have opposite signs, according the Intermediate Value Theorem, there must exist some number r such that $h(r)=0$

The function is continuous ("twice-differentiable") and because it has coordinates above and below the x-axis, there must exist some r.

Work for problem 6(a)

$\because f(2) = 5 \quad f(5) = 2$

$\therefore f'(c) = \dfrac{f(5) - f(2)}{5-2}$

$= \dfrac{2-5}{5-2} = -1$ (Mean Value Therom)

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Work for problem 6(b)

$g'(x) = f'(f(x)) \cdot f'(x)$

$g'(2) = f'(f(2)) \cdot f'(2)$

$g'(5) = f'(f(5)) \cdot f'(5)$

$f(2) = 5 \quad f(5) = 2$

$g'(2) = f'(5) \cdot f'(2)$

$g'(5) = f'(2) \cdot f'(5)$

$\therefore g'(2) = g'(5)$

$\begin{cases} g' \text{ is differentiable on interval } [2,5] \\ g'(2) = g'(5) \\ g' \text{ is contineous} \end{cases}$

$\therefore$ there is a value k for $2<k<5$ such that $g''(k) = 0$

Continue problem 6 on page 15.

AB
6B$_2$

NO CALCULATOR ALLOWED

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Work for problem 6(c)

$g' = f'(f(x)) \cdot f'(x)$ $\qquad f''(x) = 0$

$g'' = f'(x) \cdot f''(f(x)) \cdot f'(x) + f''(x) f'(f(x))$

$= (f'(x))^2 \cdot f''(f(x))$

$\because$ g'' not equal to zero

$\therefore$ the graph of g does not have a point of inflection

Work for problem 6(d)

$h(2) = f(2) - 2 = 3$

$h(5) = f(5) - 2 = 3$

and h is differentiable on $[2, 5]$

therefore, there must be a value r for $2 < r < 5$

such that $h(r) = 0$

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NO CALCULATOR ALLOWED

Work for problem 6(a)

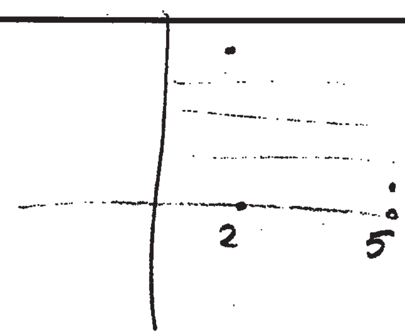
$$\frac{f(5)-f(2)}{5-2} = \frac{2-5}{3} = \frac{-3}{3} = -1$$

because the function is twice differentiable and from the mean value therom

$$\frac{f(b)-f(a)}{b-a} = f'(c)$$

$$= \frac{f(5)-f(2)}{5-2} = \frac{2-5}{3} = \frac{-3}{3} = -1 = f'(c)$$

Work for problem 6(b)



$g(2) = f(f(2))$

$g(2) = f(5) = g(2) = 2$

$g(5) = f(f(5))$

because the function is one to one function that means that the function is either decrease or increase between $(2, 5)$, and it should concave up or down and f is twice differentiable.

Continue problem 6 on page 15.

Work for problem 6(c)

$f''(x)=0$

that means the graph doesn't change in concavity, (second derivative is constant), inflection points might be found only when $f''(x)$ changes its sign.

Do not write beyond this border.

Work for problem 6(d)

$h(x)=f(x)-x$

$h(5)=f(5)-5=2-5=-3$

$h(2)=f(2)-2=5-2=3$

from Rollés Therom, we have two numbers where the function changes its sign so the must be (r) where $h(r)=0$

Do not write beyond this border.

Question 6

Sample: 6A
Score: 8

The student earned 8 points: 2 points in part (a), 3 points in part (b), 1 point in part (c), and 2 points in part (d). The student presents correct work in parts (a), (b), and (d). In part (c) the student earned the first point for considering $g''(x)$. The student makes an error in determining $g''(x)$, and so the second point was not earned. Very few students earned all 9 points.

Sample: 6B
Score: 6

The student earned 6 points: 2 points in part (a), 3 points in part (b), 1 point in part (c), and no points in part (d). The student presents correct work in parts (a) and (b). In part (c) the student correctly finds $g''(x)$ and earned the first point. The second point was not earned since the student concludes that $g''(x)$ does not equal 0. In part (d) the student does not have the correct value for $h(5)$, so the first point was not earned. Since 0 is not between the student's values of $h(2)$ and $h(5)$, the student was not eligible for the second point.

Sample: 6C
Score: 3

The student earned 3 points: 2 points in part (a), no points in part (b), no points in part (c), and 1 point in part (d). Correct work is presented in part (a). In part (b) the student writes about the function g and not g'. In part (c) the student does not refer to g''. In part (d) 1 point was earned for $h(2)$ and $h(5)$. The student appeals to Rolle's Theorem instead of the Intermediate Value Theorem, and so the second point was not earned.